Advanced Trig Day 8:

- Binomial Expansion Theorem
- Polar Average Rate of Change

HW: Binomial Expansion and Polar AROC

Warm-up!

Simplify $(A+B)^0$ $\quad 1$

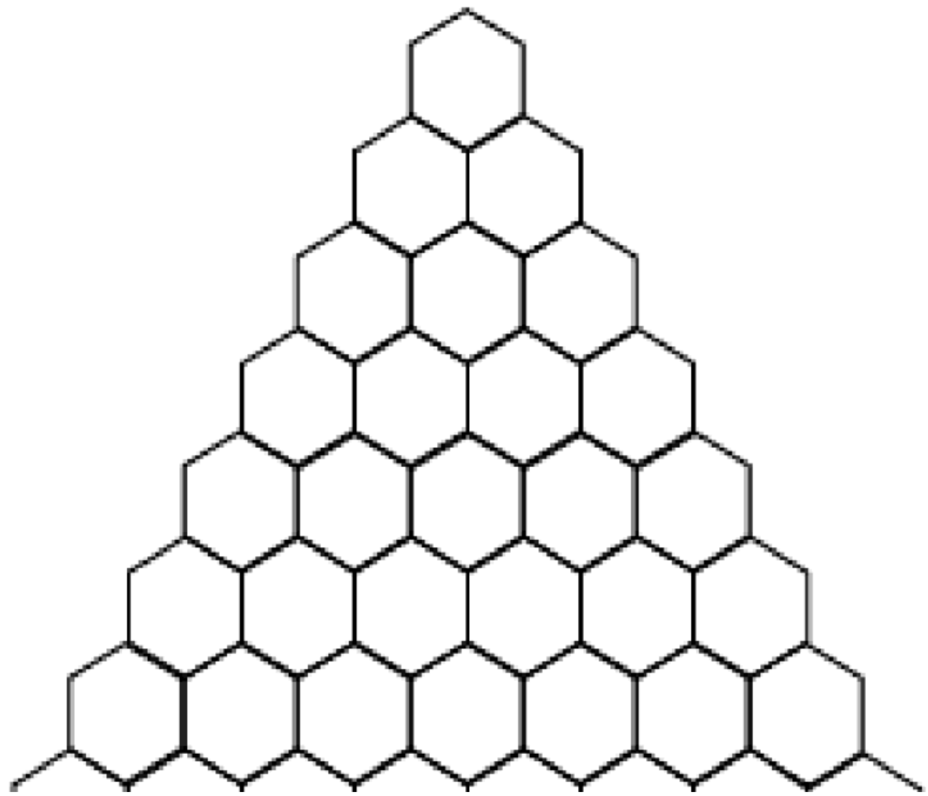
Simplify $(A+B)^1$ $\quad 1A + 1B$

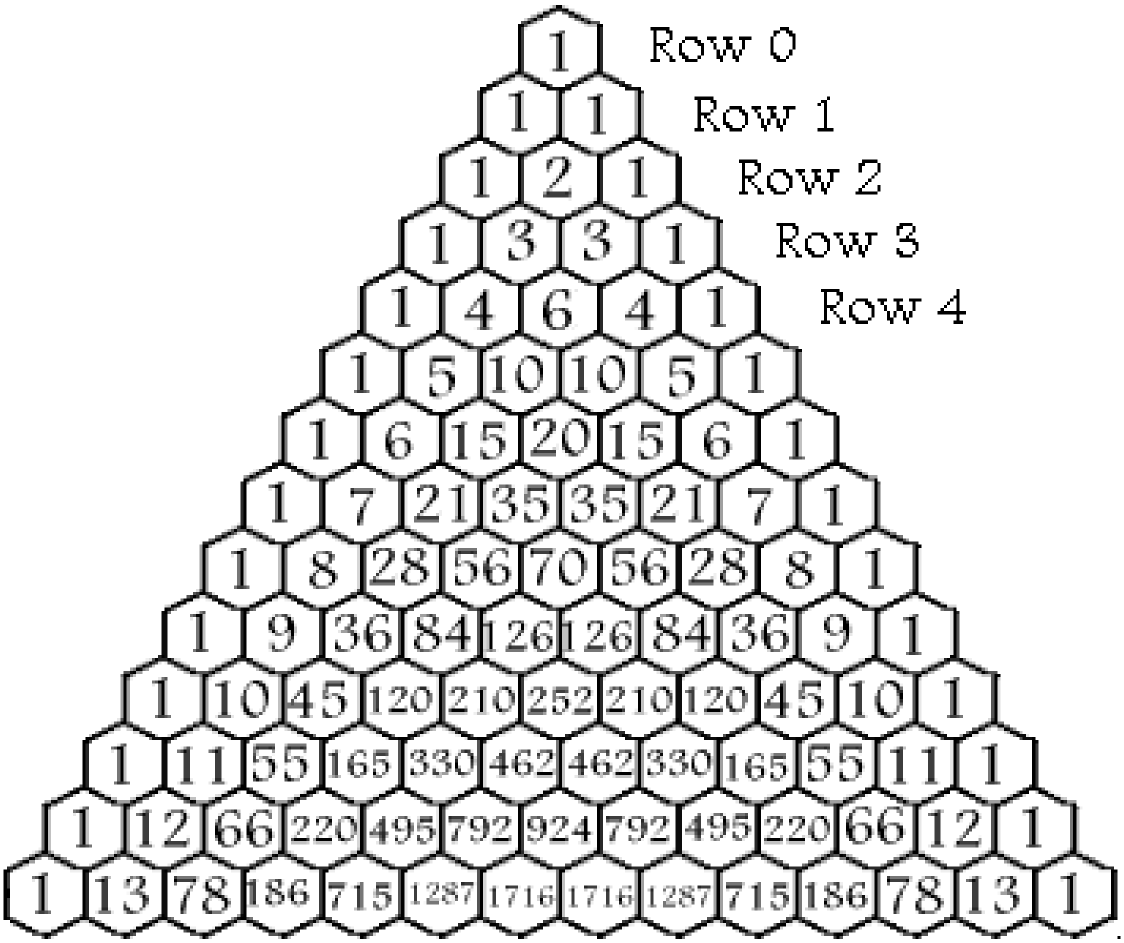
Expand $(A+B)^2$ $\quad 1A^2 + 2AB + 1B^2$

Expand $(A+B)^3$ $\quad 1A^3 + 3A^2B + 3AB^2 + 1B^3$

$(A+B)^4 = 1A^4 + 4AB^3 + 6A^2B^2 + 4AB^3 + 1B^4$

Pascal's Triangle




Row 0
Row 1
Row 2
Row 3
Row 4
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1

Take a moment to try to write Pascal's triangle on a fresh sheet of paper without looking back at what you already wrote in your notes. Try to write up to row 5 of Pascal's triangle.

Binomial Expansion Theorem

Use the Binomial Expansion Theorem to expand

$(2x - 3)^4$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$1(2x)^4(-3)^0 + 4(2x)^3(-3)^1 + 6(2x)^2(-3)^2 + 4(2x)^1(-3)^3 + 1(2x)^0(-3)^4$$

$$1\cdot 16x^4\cdot 1 \qquad 4\cdot 8x^3\cdot -3 \qquad 6\cdot 4x^2\cdot 9 \qquad 4\cdot 2x\cdot -27 \qquad 1\cdot 1\cdot 81$$

$$16x^4 + -96x^3 + 216x^2 + -216x + 81$$

Use the Binomial Expansion Theorem to expand

$$(x - 2y)^5$$

Finding a single term

Use the binomial expansion theorem to find the x^3 term in the expansion of $(2x-1)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$1(2x)^6(-1)^0 + 6(2x)^5(-1)^1 + 15(2x)^4(-1)^2 + 20(2x)^3(-1)^3 + 15(2x)^2(-1)^4 + 6(2x)^1(-1)^5 + 1(2x)^0(-1)^6$

$20(2x)^3(-1)^3$

$20 \cdot 8x^3 \cdot -1$

$-160x^3$

Use the binomial expansion theorem to find the x^4 term in the expansion of $(2x+3)^5$

$1(2x)^5(3)^0 + 5(2x)^4(3)^1 + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)^1(3)^4 + 1(2x)^0(3)^5$

$5(2x)^4(3)^1$

$5 \cdot 16x^4 \cdot 3$

$240x^4$

Harder than I think the AP exam will be, but let's be prepared for anything

Using binomial expansion theorem, find the x^6 term in the expansion of $\left(x^2 - 3\right)^5$

Last Polar Topic, I promise (it just didn't fit into the past few classes)

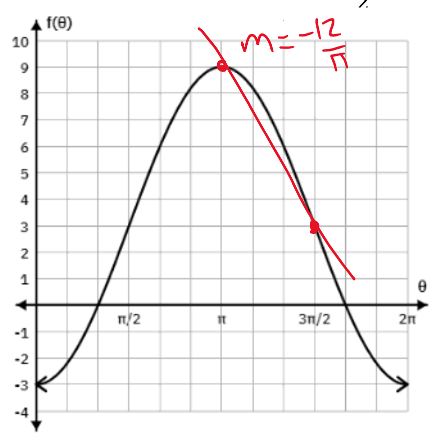
Sometimes we may be asked to find the average rate of a function $f(\theta)$ over an interval $a \leq \theta \leq b$.

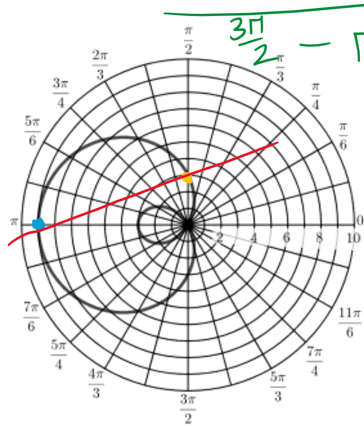
The formula for average rate of change is, as it always has been:

$$\frac{f(b) - f(a)}{b - a}$$

Example: Below is the rectangular graph of $f(\theta) = 3 - 6\cos(\theta)$ and the polar graph of $r = f(\theta)$. Find the average rate of change of $f(\theta)$ over the interval $\pi \leq \theta \leq \frac{3\pi}{2}$.

$$\frac{f(\frac{3\pi}{2}) - f(\pi)}{\frac{3\pi}{2} - \pi} = \frac{3 - 9}{\frac{3\pi}{2} - \pi} = \frac{-6}{\pi/2} = \frac{-12}{\pi}$$





$f(\frac{3\pi}{2}) = 3 - 6\cos(\frac{3\pi}{2}) = 3 - 6 \cdot 0 = 3$

$f(\pi) = 3 - 6\cos(\pi) = 3 - 6 \cdot -1 = 9$

The table below gives values of the polar function $r = f(\theta)$ at selected values of θ.

θ	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
$f(\theta)$	-1	0	2

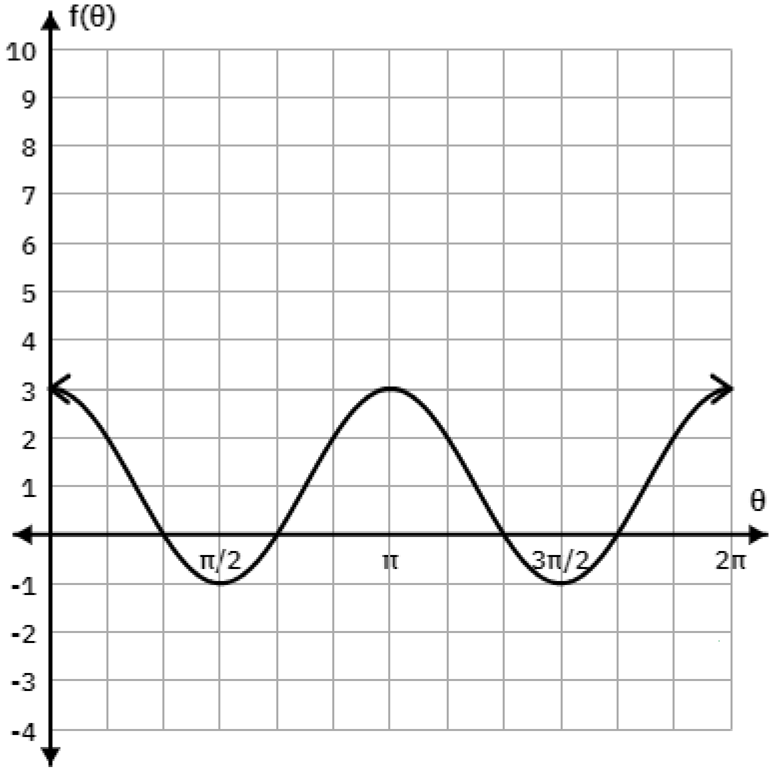
Determine the average rate of change of $f(\theta)$ on the interval $\frac{2\pi}{3}$ to $\frac{5\pi}{6}$.

$$\frac{f(b)-f(a)}{b-a} = \frac{f\left(\frac{5\pi}{6}\right)-f\left(\frac{2\pi}{3}\right)}{\frac{5\pi}{6}-\frac{2\pi}{3}} = \frac{2-0}{\frac{5\pi}{6}-\frac{2\pi}{3}} = \frac{2}{\pi/6}$$

$$= \frac{12}{\pi}$$

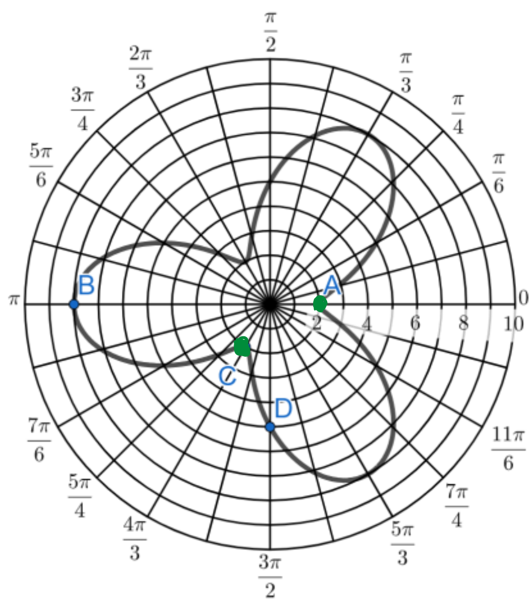
Use this average rate of change to approximate $f(\pi)$

BTW, the previous question was based off $f(\theta) = 2\cos(2\theta) + 1$



In the polar coordinate system, the graph of a polar function $r = f(\theta)$ is shown with a domain of all real values of θ for $0 \le \theta \le 2\pi$. On this interval of θ, the graph has no holes, the graph passes through each point exactly one time, and $f(\theta)$ is positive. Over which of the following intervals is the average rate of change of $f(\theta)$ zero?

$$\frac{2-2}{0-\frac{4\pi}{3}} = \frac{0}{\frac{4\pi}{3}} = 0$$



(A) $0 \le \theta \le \pi$

(B) $0 \le \theta \le \frac{4\pi}{3}$

(C) $\pi \le \theta \le \frac{3\pi}{2}$

(D) $\frac{4\pi}{3} \le \theta \le \frac{3\pi}{2}$

Expand using binomial expansion theorem

$$\left(2x - \frac{1}{2}\right)^5$$

Let's do a quick check in on our course progress and list all the AP Precalculus topics left for this year:

$$1$$
$$1 \quad 1$$
$$(A+B)^2 = 1A^2 + 2AB + 1B^2$$
$$(A+B)^3 = 1A^3 + 3A^2B + 3AB^2 + 1B^3$$
$$(A+B)^4 = 1A^4B^0 + 4A^3B^1 + 6A^2B^2 + 4A^1B^3 + 1A^0B^4$$
$$(A+B)^5 = 1A^5B^0 + 5A^4B^1 + 10A^3B^2 + 10A^2B^3 + 5A^1B^4 + 1A^0B^5$$
$$(A+B)^6 = 1A^6B^0 + 6A^5B^1 + 15A^4B^2 + 20A^3B^3 + 15A^2B^4 + 6A^1B^5 + 1A^0B^6$$